Exercise 2

(a) If A is the area of a circle with radius r and the circle expands as time passes, find dA/dt in terms of dr/dt.

(b) Suppose oil spills from a ruptured tanker and spreads in a circular pattern. If the radius of the oil spill increases at a constant rate of 1 m/s, how fast is the area of the spill increasing when the radius is 30 m?

Solution

The area of a circle with radius r is

$$A = \pi r^2.$$

Differentiate both sides with respect to t, using the chain rule on the right side.

$$\frac{d}{dt}(A) = \frac{d}{dt}(\pi r^2)$$

$$\frac{dA}{dt} = \pi \frac{d}{dt}(r^2)$$

$$\frac{dA}{dt} = \pi(2r) \cdot \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

The radius of the oil spill is increasing by 1 meter per second, so $dr/dt = 1$ m/s. Therefore, at $r = 30$ m, the rate that area is increasing is

$$\left.\frac{dA}{dt}\right|_{r=30} = 2\pi(30)(1) = 60\pi \ \frac{\text{m}^2}{\text{s}}.$$